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Oligopolistic Screening and Two-way Distortion

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Abstract
We analyze the choice of incentive contracts by oligopolistic firms that compete on the product market. Managers have private information and in the first stage they exert cost reducing effort. In equilibrium the standard "no distortion at the top" property disappears and two way distortions are optimal. We extend our analysis to other informational, contractual and competitive settings.

Keywords
Oligopoly, screening, two way distortion, incentives, RD investment

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1 Introduction

The contract theory literature has studied, in depth, how asymmetric information affects the relationship between a principal and an agent. Most of this literature has focused on isolated pairs of hierarchies, where the principal is a monopolist and the optimal contract will give some rents in excess of the reservation utility to ensure revelation and optimally solve a trade-off between incentives and efficiency by allowing some distortions away from the first-best (see for example Baron and Myerson [1982] or Stiglitz [1977]). A smaller branch of the literature has focused on principal-agent pairs, who act in a perfectly competitive market, where the zero-profit condition plays a role, but the principal makes his optimal choice of contract without external strategic element (see Rotschild and Stiglitz [1976]). Surprisingly, less has been done to analyze the effect of imperfect competition on incentives provision and most of the literature has focused on models with moral-hazard (see Hart [1983]). The aim of this paper is to study the optimal contracts, chosen by duopolists, in an hidden-information screening model.

We consider a two-stage model where, in the first stage, firm owners choose contracts for their managers, aiming to provide incentives to undertake effort in cost reducing activities; in the second stage, once uncertainty on contracts, efforts and costs has been resolved, firms engage in some form of product market competition. In our baseline model, we focus on quantity competition with substitutable goods, where the efforts of the two managers are strategic substitutes, in the sense that a higher effort by one reduces the marginal profitability of effort of the other firm. Managers differ in their disutility of effort and their types are independently distributed.

We first consider a first-best benchmark case of contract competition under uncertainty where the productivity of an agent can be observed by his principal but not by the rival. We assume a Nash behavior in the contract offer, that is, effort/wage pairs are chosen simultaneously, taking as given those offered by the other firm. The uncertainty about the manager's type in the rival hierarchy produces by itself a strategic effect that makes interdependent the optimal level of efforts for the different types (this is due to the fact that the marginal benefit of effort differs, not only with one's own manager's type, but also with the rival manager's type[1]). Once we move to the asymmetric information setup, where each agent's type is private information to him, we observe that the informational rent paid to high types, coupled with the strategic effect of the competing contract, eliminates the "no distortion at the top" property and a "two-way" distortion

[1] In our companion paper, Etro and Cella [2010], the analysis is extended to a continuum of types and n-firms and we show that the optimal level of effort for each type depends on the whole distribution of types.

becomes optimal. In other words, the equilibrium screening contract prescribes a level of effort in excess of the first-best level for efficient managers and one below the first best level for inefficient managers.

The "two-way" distortion is robust to all our extensions, where effort levels are strategic substitutes: when types are imperfectly correlated, when contracts between firms and managers include quantity commitments, and when firms produce differentiated goods and compete in prices rather than in quantities. The strategic interaction in the choice of contracts leads firms to polarize their requests from different types, with higher effort asked to the good managers (more likely to face bad ones) and *vice-versa*. However, in those extensions where efforts are strategic complements their equilibrium levels will be downward distorted with respect to the first best for all types: this happens in case of complement goods produced by the firms or in case of investments in advertising which enhances demand. Therefore, the kind of strategic interactions between the efforts of the managers (and not the kind of competition) leads to different consequences on the equilibrium contracts.

This work belongs to the branch of literature that studies the influence of competition on incentive mechanisms. Most of the studies have focused on the issue of moral hazard and have shown how competition reduces profits and, therefore, the marginal benefits of effort. Raith [2003] identifies this effect, together with a positive effect on incentives coming from an increase in demand elasticity due to competition. This scale effect is present also in most hidden information models, e.g. Martin [1993][2] that finds a negative effect of competition on efforts due to a scale effect. Schmidt [1997] studies cost reduction, within a moral hazard framework and a very stylized market game, where he observes that the value of cost reduction depends on the efficiency of the other duopolist, exactly like in our model.

Some hidden information models have analyzed a setting where duopolists engage in price discrimination, generating problems of common agency (see Ivaldi and Martimort [1994]) within a significantly different context than ours. The most relevant article on screening within an oligopolistic framework is Martimort [1996], that compares the profitability of exclusive dealing versus a common retailer (a problem of common agency). To analyze the exclusive dealing case, the author develops a model that allows the analysis of competition through secret contracts. His main finding is a competing contract effect, that reduces the distortion generated by the standard rent-extraction/efficiency trade-off when goods are substitutes. In his model, the contract offered by the rival firm affects the agent's incentive constraints directly, therefore modifying the marginal cost

[2] Bertoletti and Poletti [1996] is extremely useful in understanding Martin's result.

of inducing effort. In our model, the rival firm's contract affects the objective function of the principal directly and modifies the marginal benefit of effort. A similar result is obtained by Brainard and Martimort [1996], that considers the effect of asymmetric information on strategic trade policy, where principal agent hierarchies compete through public contracts. We also have public contracts, but in their framework private information is perfectly correlated. Recently, Piccolo, D'Amato and Martina [2008] have studied the relationship between product market competition and organizational slack, under different contractual regimes. Assuming perfectly correlated types, when contracts are conditioned on costs, competition has no effect on the firms' internal agency problem. In that case, incentives are in fact independent from the rival's performance and only a scale effect is present. They also show that, if profits are used to control managerial behavior, then competition has a direct impact on managers' incentives. A competing-contract effect (as in Martimort [1996]) mitigates the agency conflict. All the three papers above show that, however mitigated by competition the agency conflict inside the hierarchy is solved in a familiar way, with no distortion at the top and downward distortions for all but the most efficient types.

Our findings may be reminiscent of the countervailing incentives literature (see Lewis and Sappington [1989] and Maggi and Rodriguez-Clare [1995] where an agent's incentive to overreport or underreport depend on his type. In our model an agent has always an incentive to overreport his disutility from effort.[3] Two-way distortion makes an appearance also in a principal multi-agent model in Lockwood [2000] due to a production externality.

The paper is organized as follows. Section 2 introduces the model and assumptions. Section 3 analyzes two benchmarks. Section 4 presents our first best solution. Section 5 contains the main result. Section 6 extend to the case of positive correlation. Section 7 uses more comprehensive contracts. Section 8 studies the case of complement goods. Section 9 presents two further extensions. Section 10 concludes.

2 The model

Consider two firms, i and j, that operate in a market with inverse demand $p = a - X$, where X is total quantity produced and a is a size parameter. Production requires a constant marginal cost which can be reduced by a manager's effort. For simplicity, we

[3] More importantly, our two way distortion disappears if we consider a hierarchy in isolation as we go back to a more standard monopolistic screening framework.

assume that effort e generates the marginal cost $c(e) = c - \sqrt{e}$.[4]

The manager's utility function is:

$$u(w, e) = w - \theta_k e, \tag{1}$$

where θ_k, with $k = 1, 2$, is the marginal disutility of effort of the manager. It is privately known and is a random variable with discrete support $[\theta_1, \theta_2]$ with $0 < \theta_1 < \theta_2$, $\Delta\theta = \theta_2 - \theta_1$ and $\Pr(\theta = \theta_1) = \nu$. We make the further assumption that that θ is i.i.d across managers.

Effort and its disutility are not observable while realized marginal costs are verifiable (by own principal), these assumptions place our analysis in a traditional screening framework.

Firm owners offer a contract to their respective managers to induce cost reducing effort. A contract establishes the size of cost reduction, or equivalently the effort e, and a wage w.

When offering the contract, each firms takes the contract offered by the rival firm as given. In our setting a contract ensures participation and truthful revelation by the manager but it is also a best response to the cost reducing activity that the other firm does through her own contract.

Contract offers are made simultaneously and realized costs become observable (but not verifiable) by everybody at the end of the first period. This modelling assumption has two consequences: first of all, second stage product competition happens in a world without uncertainty[5] and, second, contracts are, in a way, necessarily incomplete in the sense that one firm cannot condition his own contract on the type of the other firm's manager.[6]

The timing of our game is as follows:

1. Nature draws θ_k^i and θ_k^j;
2. Both firms invest in cost reduction by offering a contract to their managers;
3. Managers accept or reject the contract;
4. Managers report their own type (choose a wage/effort pair);
5. Investments and payment take place;

[4] None of our qualitative results depend on the functional form of the cost function, that could be easily generalized to $C(\theta, e)$ with $C_e > 0$, $C_{ee} \geq 0$, $C_\theta > 0$ and $C_{e\theta} > 0$, the latter being the Spence-Mirrlees condition.

[5] This makes our analysis more tractable analytically, but our qualitative results would not change if contracts and costs remained secret.

[6] This contracting limitation may be due to problems of verifiability on the other firm's costs for lack of auditing rights and abilities (see Brainard and Martimort [1996])

6. Marginal costs become public;
7. Firms compete à la Cournot in the product market.

3 Benchmarks

This section briefly presents some benchmark solutions: the monopoly case and the full information Cournot model.

3.1 The monopolist solution.

At the second stage of product market competition a profit maximizing monopolist will produce $x_M = [a - c + \sqrt{e}]/2$ and obtain profits equal to $\pi^M = [a - c + \sqrt{e}]^2/4 - w$. If the manager has no private information about his type, the firm owner will choose the contract (e^{*M}, w^{*M}) that maximizes profits under the participation constraint $w^{*M} = \theta_k e^{*M}$. The first best contract requires $\sqrt{e_k^{*M}} = (a - c) / (4\theta_k - 1)$. This effort is also the one that minimizes total costs (effort provision and production costs) at the monopoly output[7].

In the case of private information the principal/owner will have to satisfy incentive compatibility constraints to ensure truthtelling from the manager/agent. Informational rents for the most efficient type of manager will require downward distortion in the level of effort required to the less efficient manager, namely $\sqrt{e_1^M} = (a - c) / (4\theta_1 - 1)$ and $\sqrt{e_2^M} = (a - c) / \left(4\left(\theta_2 + \frac{\upsilon}{1-\upsilon}\Delta\theta\right) - 1\right)$. This is the well known result of standard screening models, the most efficient manager will have to exert an efficient level of effort (the "no distortion at the top" property) while any other type's effort is reduced below the first best level.

3.2 The Cournot solution.

We now present the analysis of a duopoly in which two firms i and j choose their contracts to maximize profits under a participation constraint for their managers and taking as given the contracts of each other, there is no private information and the managers' types are common knowledge. Given two contracts (e^i, w^i) and (e^j, w^j), at the second stage of product competition, firm i obtains the following profits:

$$\pi^i = (a - x_i - x_j)x_i - \left(c - \sqrt{e^i}\right) x_i - w_i \tag{2}$$

[7] If total costs are $TC(\theta, x, e) = (c - \sqrt{e})\, x + \theta e$ then the effort that minimizes total costs for any given output is $\sqrt{e_k{}^{\circ}} = \frac{x_k}{2\theta_k}$.

In a Cournot equilibrium each duopolist produces:

$$x_i = \frac{a - c + 2\sqrt{e^i} - \sqrt{e^j}}{3} \tag{3}$$

and the equilibrium price is:

$$p = \frac{a + 2c - \sqrt{e^i} - \sqrt{e^j}}{3} \tag{4}$$

Accordingly, the first stage profits of firm i can be expressed as

$$\pi^i = \Pi(e^i, e^j) - w^i \tag{5}$$

where:

$$\Pi(e^i, e^j) = \frac{\left(a - c + 2\sqrt{e^i} - \sqrt{e^j}\right)^2}{9} \tag{6}$$

and the optimal contract maximizes these profits under the participation constraint $w^i = \theta e^i$. Notice that the effort levels are strategic substitutes:

$$\Pi_{12}(e^i, e^j) = -\frac{1}{9\sqrt{e^i e^j}} < 0 \tag{7}$$

therefore a higher effort by one manager reduces the marginal profitability of effort for the other firm.

When both managers are of type θ, the optimal contract of firm i must satisfy the condition $\sqrt{e^i} = 2(a - c - \sqrt{e^j}) / (9\theta - 4)$, and the same condition holds for the other firm. The symmetric Nash equilibrium levels of effort required will be $\sqrt{e^*} = 2\,(a - c)\,/\,(9\theta - 2)$, which is lower than the one under monopoly. As it is well known, competition reduces prices and profits, but does not increase effort. This is due to a scale effect (see for example Martin [1993]) caused by the fact that in a duopoly model each firm is facing a lower residual demand and the marginal benefit of effort is smaller.

It is worth noting that although effort decreases it is still higher than the level that would minimize total costs. The reason is to be found in the Cournot type of competition coupled with the two stages set-up, as pointed out by Brander and Spencer [1983]. They show that, when investment in cost reduction is made before the associated output is produced, firms tend to shift resources to the first stage so that marginal costs are lower and they can gain an advantage in the imperfectly competitive output game.

This simple model of contract competition in presence of identical managers can be extended in many ways by altering the informational structure. In the subsequent sections we will make the assumption that the types of the managers are independently distributed. This setup will allow us to study how a principal will modify his contract offer when he knows his rival will adopt a similar behavior in presence of manager's specific shocks.[8]

4 Contract competition with symmetric information

We now assume that, at the contract offer stage, each firm knows the type of its own manager but not that of the other firm, and can condition its contract only on the former. In other words, there is uncertainty on the type of the other firm's manager, but there is no asymmetric information in this framework. We can consider the contracts offered in this setup as the first best benchmark of our oligopolistic screening framework.

Contracts are chosen simultaneously taking as given those offered by the other firm. At the second stage, uncertainty is resolved and production decisions take place simultaneously knowing the true realized costs of each firm.

We solve the game by backward induction. Given two contracts (e^i, w^i) and (e^j, w^j), the two firms produce as in (3) and obtain profits as in (6).

The optimal contract, (e^i_k, w^i_k), for each firm i and with a manager of type k, will maximize expected profits subject to a participation constraint, namely:

$$\begin{aligned} \max_{e^i_k, w^i_k} E(\pi^i_k) &= \qquad (8) \\ &= \nu \frac{\left(a - c + 2\sqrt{e^i_k} - \sqrt{e^j_1}\right)^2}{9} + (1-\nu)\frac{\left(a - c + 2\sqrt{e^i_k} - \sqrt{e^j_2}\right)^2}{9} - w^i_k \\ \text{s.t. } w^i_k &= \theta_k e^i_k \qquad \text{with } k = 1, 2 \end{aligned}$$

The above expectation is just an average of the profits earned when competing with a rival who employs an efficient or inefficient manager. The participation constraint is binding and the optimal effort of a manager of type θ_k in firm i satisfies the following first order condition:

[8] The perfect correlation of many related works fits better the idea of idiosyncratic demand shocks.

$$\sqrt{e_k^i} = \frac{2\left(a - c - \nu\sqrt{e_1^j} - (1-\nu)\sqrt{e_2^j}\right)}{9\theta_k - 4} \quad j, k = 1, 2 \text{ and } j \neq i \tag{9}$$

The following Proposition characterizes the symmetric equilibrium of our model.

Proposition 1 *When each principal can observe the type of his own manager the optimal level of efforts are given by the following equilibrium conditions:*

$$\sqrt{e_1^*} = \frac{2\left(a - c - (1-\nu)\sqrt{e_2^*}\right)}{9\theta_1 + 2\nu - 4} \tag{10}$$

$$\sqrt{e_2^*} = \frac{2\left(a - c - \nu\sqrt{e_1^*}\right)}{9\theta_2 + 2(1-\nu) - 4} \tag{11}$$

Moreover $\sqrt{e_1^*/e_2^*} > \sqrt{e_1^{*M}/e_2^{*M}}$*, implying that competition leads to a polarization of equilibrium efforts.*

Proof. Taking (9) and imposing symmetry gives immediately the two equilibrium conditions (10) and (11). By solving the system of two equations we get the optimal level of effort:

$$\sqrt{e_k^*} = \frac{2(a-c)\left(9\theta_z - 4\right)}{\left(9\theta_1 + 2\nu - 4\right)\left(9\theta_2 - 2\nu - 2\right) - 4\nu(1-\nu)} \quad \text{for } k, z = 1, 2 \text{ and } k \neq z \tag{12}$$

The ratio of efforts is $\sqrt{e_1^*/e_2^*} = (9\theta_2 - 4)/(9\theta_1 - 4)$ that is unambiguously higher than what we had in our benchmark monopoly case $\sqrt{e_1^M/e_2^M} = (4\theta_2 - 1)/(4\theta_1 - 1)$. ■

Competition generates a tendency toward polarization of efforts. To see how this happens, notice that in general firms induce a high effort when they have an efficient manager because the marginal return of effort is higher, this return is even larger when the probability of meeting an inefficient rival is high (ν is low). They, instead, induce a very low effort when they have an inefficient manager, especially if they are likely to meet an efficient rival (ν is high). Competition increases the distance between the two equilibrium contracts because a lower effort of the inefficient rival increases the incentives to exert effort for an efficient manager - see (10). Moreover, a higher effort of the efficient rival leads to a reduction of the effort of an inefficient manager - see (11) - because it reduces the marginal return from effort, especially when facing a more efficient rival. Therefore, competition with uncertainty on the rivals' types leads to an increase in the ratio between the effort required from an efficient manager and that required from an inefficient manager.

In this setup where there is no asymmetric information inside the hierarchy we already observe an effect of the strategic interaction in the product market on the provision of incentives. The commitment effect of deciding costs at the first stage coupled with the strategic effect of the other hierarchy's contract offer modifies even further the marginal benefit on inducing effort.

In other words, competition in the product market leads the principal to offer contracts that are also a best response to the contractual behavior of the other principal, this strategic element produces optimal efforts whose ratio is higher than in the monopoly setting.

This strategic effect can be so large to completely reverse the scale effect we have observed in our benchmark Cournot model. One can, in fact, verify that the effort required from the efficient manager can be larger, in absolute value, than the one required by a monopolist with a manager of the same type, $\sqrt{e_1^M} = (a-c)\,/\,(4\theta_1 - 1)\,.$[9] However, when θ_2 is high enough, we obtain a stronger outcome: even the weighted average effort $\nu\sqrt{e_1^*} + (1-\nu)\sqrt{e_2^*}$ is larger under duopoly compared to the average effort under monopoly.[10] In other words, when managers are more likely to be inefficient, competition induces firms with efficient managers to exert more effort than if they were monopolists, and when the productivity differential is large enough competition increases the average effort as well.

This increase in effort has the obvious direct consequence of reducing marginal production costs and having more "aggressive" firms compete in the product market. Overall this is still far from saying that firms increase their efficiency, if with that we mean operating at the minimum of total costs. Since effort is remunerated at the first stage (and then those costs are sunk at the second one), there is a built in tendency to have the manager exert too much cost reducing activity. To put it simply firms are prone to be inefficient because they put too much effort, not too little. Consumers obviously gain from this type of inefficiency.

[9] This happens for ν and θ_1 small enough and θ_2 large enough. Assume $\nu = 0.5$: then, one can verify that $e_1^* > e_1^M$ if $\theta_1 < 1$ and $\theta_2 > 5\theta_1/9(1-\theta_1)$.

[10] For instance, with $a-c = 1$, $\theta_1 = 2/3$ and $\theta_2 = 10$, we have $\sqrt{e_1^M} = 3/5$, $\sqrt{e_2^M} = 1/39$, $\sqrt{e_1^*} = 43/65$ and $\sqrt{e_2^*} = 1/65$ and the average effort is 0.3 in monopoly and 0.34 in duopoly.

5 Contract competition with asymmetric information

In this section we make the assumption that managers have private information about their cost reduction ability. When offering a contract each principal will optimally screen for its own manager's type and will take as given the optimal contractual behavior of the rival firm.[11] Since the second stage product market competition is unaffected the optimal quantity is still given by (3) and profits take the form of (6).

The firm owners now maximize their expected profits that take the form:

$$\max_{(e_k^i, w_k^i)} E(\pi^i) = \tag{13}$$

$$= \nu \left[\nu \frac{\left(a - c + 2\sqrt{e_1^i} - \sqrt{e_1^j}\right)^2}{9} + v(1-\nu) \frac{\left(a - c + 2\sqrt{e_1^i} - \sqrt{e_2^j}\right)^2}{9} - w_1^i \right] +$$

$$(1-\nu) \left[\nu \frac{\left(a - c + 2\sqrt{e_2^i} - \sqrt{e_1^j}\right)^2}{9} + (1-\nu) \frac{\left(a - c + 2\sqrt{e_2^i} - \sqrt{e_2^j}\right)^2}{9} - w_2^i \right]$$

$$\text{s.t.} : w_1^i \geq \theta_1 e_1^i \text{ and } w_2^i \geq \theta_2 e_2^i$$
$$w_1^i - \theta_1 e_1^i \geq w_2^i - \theta_1 e_2^i \text{ and } w_2^i - \theta_2 e_2^i \geq w_1^i - \theta_2 e_1^i$$

where (e_k^i, w_k^i) is the contract chosen by firm i for its manager of type θ_k with $k = 1, 2$. The first pair of constraints ensure participation while second one will guarantee truthtelling. Standard arguments imply that the binding constraints are the participation constraint for the inefficient manager and the incentive compatibility constraint for the efficient one[12], this allows us to derive the individually rational and incentive compatible wages:

$$w_2^i = \theta_2 e_2^i \text{ and } w_1^i = \theta_1 e_1^i + \theta_2 e_2^i - \theta_1 e_2^i \equiv \theta_1 e_1^i + \Delta\theta e_2^i. \tag{14}$$

Using these constraints, the optimal contract for firm i with a manager of type θ_k satisfies

[11] As mentioned before we assume that firm i's contract cannot be conditioned on the type of the manager of firm j.

[12] We will check later that the solution is monotonic, that will guarantee global incentive compatibility. There is no need for a modified monotonicity condition as in Piccolo *et al.* [2008] because incentives constraint are not modified by contract competition.

the following first order condition:

$$\sqrt{e_k^i} = \frac{2\left(a - c - \nu\sqrt{e_1^j} - (1-\nu)\sqrt{e_2^j}\right)}{9\tilde{\theta}_k - 4} \tag{15}$$

where $\tilde{\theta}_1 = \theta_1$ and $\tilde{\theta}_2 = \theta_2 + \frac{v}{1-v}\Delta\theta$. In a symmetric Perfect Bayesian equilibrium in the choice of contracts, it must be that both firms choose the same contracts $(e_1, w_1) = (e_1, \theta_1 e_1 + \Delta\theta e_2)$ and $(e_2, w_2) = (e_2, \theta_2 e_2)$.

We can now characterize our equilibrium screening contracts.

Proposition 2 *When managers have private information about their type the optimal level of efforts are given by the following equilibrium conditions:*

$$\sqrt{e_1} = \frac{2\left(a - c - (1-\nu)\sqrt{e_2}\right)}{9\theta_1 + 2\nu - 4} \tag{16}$$

$$\sqrt{e_2} = \frac{2\left(a - c - \nu\sqrt{e_1}\right)}{9\left(\theta_2 + \frac{v}{1-v}\Delta\theta\right) + 2(1-\nu) - 4} \tag{17}$$

Moreover $e_1 > e_1^*$ *and* $e_2 < e_2^*$*, meaning that both types exert inefficient levels of efforts.*

Proof. Taking (15) and imposing symmetry gives immediately the two equilibrium conditions (16) and (17). By solving the system of two equations we get the optimal level of effort:

$$\sqrt{e_k} = \frac{2(a-c)\left(9\tilde{\theta}_z - 4\right)}{(9\theta_1 + 2\nu - 4)\left[9\left(\theta_2 + \frac{v}{1-v}\Delta\theta\right) - 2\nu - 2\right] - 4\nu(1-\nu)} \text{ for } k, z = 1, 2 \text{ and } k \neq z \tag{18}$$

and where $\tilde{\theta}_1 = \theta_1$ and $\tilde{\theta}_2 = \theta_2 + \frac{v}{1-v}\Delta\theta$.

Then note that (16) is the same as (10) while (17) is different from (11) because at the denominator we have the virtual type of the inefficient manager. As a consequence the second best value for e_2 will be lower than the first best case while e_1 will be higher. ■

This has shown a crucial feature of contract competition and oligopolistic screening: contrary to what happens in the case of monopolistic screening, the equilibrium effort of the efficient manager depends on the equilibrium effort of the inefficient one and, when informational rents have to be paid to ensure revelation, the no distortion at the top property disappears. The strategic effect discussed previously and asymmetric information within the hierarchy imply that the contract requires inefficient efforts from both, with efficient managers asked to provide more effort than in the first best ($e_1 > e_1^*$) and inefficient ones asked to provide less ($e_2 < e_2^*$). This brings to an additional polarization of the effort levels. The fact that a firm provides low incentives to an inefficient manager to insure incentive compatibility forces the other firm to require extra effort from the efficient manager to exploit the higher return from effort (especially in case the rival is inefficient). In turn, stronger incentives for the efficient managers reduce the marginal return of effort of the inefficient rivals even further. The two mechanisms reinforce each other due to the strategic element in the contracts choice.

In conclusion, asymmetric information increases the equilibrium effort of the efficient manager, while reducing his informational rent, and reduces the equilibrium effort of the inefficient manager.

This has immediate implications for the "strength" of the incentive contracts, which can be measured through the ratio between different efforts or, equivalently, with the ratio between wages for efficient and inefficient managers, that we can derive from (14) as:

$$\frac{w_1}{w_2} = \frac{\theta_1}{\theta_2}\left(\frac{e_1}{e_2}\right) + \frac{\Delta}{\theta_2}$$

The effort differential can is $\sqrt{e_1/e_2} = \left(9\left(\theta_2 + \frac{v}{1-v}\Delta\theta\right) - 4\right)/(9\theta_1 - 4)$ that is higher than $\sqrt{e_1^*/e_2^*}$ and as it was the case in the previous section, the effort required from the efficient manager can be larger in absolute value than the one required by a monopolist.[13]

This example also grants us the possibility of looking at the consequences of contract competition on the market structure, namely on profits and prices. Denoting with Π_{ij} the gross profits of a firm with a manager of type i competing against one with a manager of type j, from (6) we have the following final gross profits:

$$\Pi_{12} = \frac{\left(a - c + 2\sqrt{e_1} - \sqrt{e_2}\right)^2}{9} > \Pi_{11} = \frac{\left(a - c + \sqrt{e_1}\right)^2}{9} >$$

$$> \Pi_{22} = \frac{\left(a - c + \sqrt{e_2}\right)^2}{9} > \Pi_{21} = \frac{\left(a - c + 2\sqrt{e_2} - \sqrt{e_1}\right)^2}{9}$$

Asymmetric information definitely increases the volatility of the gross profits, because e_1 (e_2) goes up (down) compared to e_1^* (e_2^*). However, results are less clear-cut for the net profits.

We can also look at the equilibrium price that is always given by (4) depending on

[13] Again, this happens for ν and θ_1 small enough and θ_2 large enough, and when θ_2 is high enough also the average effort is larger under duopoly compared to the monopoly with asymmetric information.

the types of the managers, with an expected value given by:

$$E_\theta[p] = \frac{a + 2(c - \nu\sqrt{e_1} - (1-\nu)\sqrt{e_2})}{3} \tag{19}$$

It is easy to verify that asymmetric information reduces the expected price (the weighted effort goes down when we introduce a distortion), but price dispersion increases. On one side asymmetric information leads to a price reduction in case of two efficient managers compared to the outcome without asymmetric information: the corresponding price $p_{11} = a/3 + (2/3)\left(c - \sqrt{e_1}\right)$ goes down since e_1 goes up compared to e_1^*. The opposite occurs in case of two inefficient managers, since asymmetric information gives rise to a higher price in this case: $p_{22} = a/3 + (2/3)\left(c - \sqrt{e_2}\right)$ goes up. The mixed cases in which one firm only has an efficient manager produces an ambiguous price change ($p_{12} = p_{21} = a/3 + \left(2c - \sqrt{e_1} - \sqrt{e_2}\right)/3$). However, when $\nu = 1/2$ this price is the same as the expected price and in such a case we can conclude that price volatility increases because of asymmetric information.

6 Correlated types

In this section we extend the basic duopolistic model to generally correlated types. The main finding is that negative correlation tends to enhance polarization of the effort levels of high and low productivity managers, while positive correlation tends to reduce the polarization. In the limit case of perfect correlation we return to the no-distortion at the top result, as in Martimort (1996).

The intuition for the latter result is pretty straightforward: if correlation is perfect, the principal, when offering a contract, knows that, whatever the type of his own manager, the type of the other firm's manager will be the same. The strategic effect that in the case of independently distributed types resulted in extra effort for high types disappears. The effort equilibrium conditions for the two types are not interdependent and the downward distortion on the less efficient manager's effort has no consequence on the effort required to the most efficient one. In this case there is really no interaction between the contract offers and the most efficient manager exerts a first best level of effort.

Let's assume then that the joint probability distribution is given by the following: $p_1 = \Pr\left(\theta^i = \theta_1 \text{ and } \theta^j = \theta_1\right)$, $p_2 = \Pr\left(\theta^i = \theta_2 \text{ and } \theta^j = \theta_2\right)$ and $\frac{\widehat{p}}{2} = \Pr\left(\theta^i = \theta_1 \text{ and } \theta^j = \theta_2\right) = \Pr\left(\theta^i = \theta_2 \text{ and } \theta^j = \theta_1\right)$. We are in presence of positive correlation if $\rho = p_1 p_2 - \frac{\widehat{p}^2}{4} > 0$. The following Proposition describes the optimal effort behavior in presence of correlation.

Proposition 3 *When the type of the manager is known to its own principal and types are positively correlated the ratio of optimal efforts can be written as:*

$$\sqrt{\frac{e_1^*}{e_2^*}} = \frac{9\theta_2 - 4 + d^2 + d + 4\rho}{9\theta_1 - 4 + d^2 - d + 4\rho}$$

where $d = p_2 - p_1$.

This expression is always decreasing in ρ *if* $d > 0$ *and when* $d < 0$ *it is decreasing if* $\Delta\theta > \frac{2}{9}d$.

Proof. Because p_1, p_2 and $\frac{\widehat{p}}{2}$ are probabilities we can write $\widehat{p} = 1 - p_1 - p_2$ and we can substitute this expression in ρ that becomes:

$$\rho = p_1 p_2 - \frac{[1 - (p_1 + p_2)]^2}{4} \tag{20}$$

. Let $d = p_2 - p_1$ and substitute in (20):

$$\rho = p_1 - \left(\frac{d^2}{4} + \frac{1}{4} - \frac{d}{2}\right)$$

that can be rewritten as:

$$4p_1 = 4\rho + \left(d^2 + 1 - 2d\right)$$

In case of correlation, and with the joint probability distribution given above, the optimal efforts of section 4 become:

$$\sqrt{e_1^*} = \frac{2(a-c)(9\theta_2 + 2p_2 - \widehat{p} - 4)}{(9\theta_1 + 2p_1 - 4)(9\theta_2 + 2p_2 - 4) - \widehat{p}^2}$$

$$\sqrt{e_{12}^*} = \frac{2(a-c)(9\theta_1 + 2p_1 - \widehat{p} - 4)}{(9\theta_1 + 2p_1 - 4)(9\theta_2 + 2p_2 - 4) - \widehat{p}^2}$$

If we substitute for $\widehat{p}, d$ and ρ, their ratio becomes:

$$\sqrt{\frac{e_1^*}{e_2^*}} = \frac{9\theta_2 - 4 + d^2 + d + 4\rho}{9\theta_1 - 4 + d^2 - d + 4\rho}$$

and

$$\left.\frac{\partial\sqrt{\frac{e_1^*}{e_2^*}}}{\partial\rho}\right|_d = -\frac{4[9(\theta_2 - \theta_1) + 2d]}{[9\theta_1 - 4 + d^2 - d + 4\rho]^2}$$

When $d > 0$ the above is always negative. Remember that $-1 < d < 0$, so the above derivative is negative whenever $\Delta\theta > \frac{2}{9}|d|$. ■

We have seen how positive correlation weakens the strategic effect described in the previous section by reducing the effort differential. Nonetheless equilibrium efforts remain interdependent unless positive correlation is perfect, implying that two way distortions persist once asymmetric information is considered.

When correlation is negative our strategic effect is reinforced so that the effort differential increases with respect to our main example where types were independently distributed. Equilibrium interdependence of efforts and two way distortions persist even when negative correlation is perfect.

7 Contracts with quantity commitments

In this section we analyze contract competition in duopoly when a contract includes not only a wage and an effort choice but also an output level for each state of the world. This is equivalent to the case considered by Martimort (1996) and Piccolo *et al.* (2008) in their models with perfectly correlated types. As one would expect, the availability of a more comprehensive contract reduces the equilibrium effort but does not change the qualitative nature of our results: two way distortions remain and no-distortion at the top disappears.

Consider the model of the previous sections with the difference that a contract for a manager of type θ_k with $k = 1, 2$ is now the vector (e_k^i, x_k^i, w_k^i) that specifies effort, production and wage. In other words the two-stage game is compressed into one stage and firm i (paired with a manager of type $k = 1, 2$) solves following optimization problem:

$$\begin{aligned} \max_{(e_k^i, x_k^i, w_k^i)} E(\pi_k^i) &= \left(a - c + \sqrt{e_k^i} - x_k^i - \nu\left(x_1^j\right) - (1-\nu)\left(x_2^j\right)\right) x_k^i - w_k^i \\ \text{s.t.} \quad w_k^i &\geq \theta_k e_k^i \end{aligned} \tag{21}$$

where x_1^j (resp. x_2^j) is the quantity produced by the rival firm when her manager is efficient (resp. inefficient). The only constraints are the individual rationality ones, as the manager does not have private information regarding his type but profits depend from the unknown type of the other firm's manager. The principal will compute the optimal contract taking as given the contract of the other firm. The first order conditions for firm i can be rewritten as:

$$\begin{aligned}
\sqrt{e_k^i} &= \frac{x_k^i}{2\theta_k} \\
x_k^i &= \frac{(a-c)+\sqrt{e_k^i}-\nu\left(x_1^j\right)-(1-\nu)\left(x_2^j\right)}{2} \\
w_k^i &= \theta_k e_k^i \qquad\qquad k=1,2
\end{aligned}$$

The optimal level of effort happens to be the one that minimizes total costs for any level of output, in fact once marginal costs are determined together with output decisions the commitment effect that was causing inefficiently high effort disappears.

Since analogous conditions hold for firm j. we can impose symmetry and obtain the following equilibrium production levels:

$$x_1 = \frac{(a-c)+\sqrt{e_1}-(1-\nu)(x_2)}{2+\nu} \tag{22}$$

$$x_2 = \frac{(a-c)+\sqrt{e_2}-\nu(x_1)}{2+1-\nu} \tag{23}$$

These equilibrium conditions show that once quantities are set at the first (and only) stage of the game we then have interdependence between the quantity produced by a firm with an efficient manager and that produced a by a firm with an inefficient one. As the following Proposition shows the property can be found in optimal effort levels as well.

Proposition 4 *When a contract includes output decisions and each principal can observe the type of his own manager the optimal level of efforts are given by the following equilibrium conditions:*

$$\sqrt{e_1^{**}} = \frac{2(a-c)-(1-\nu)\sqrt{e_2^{**}}}{12\theta_1-2-(1-\nu)} \tag{24}$$

$$\sqrt{e_2^{**}} = \frac{2(a-c)-\nu\sqrt{e_1^{**}}}{12\theta_2-2-\nu} \tag{25}$$

Moreover $\sqrt{e_1^{**}/e_2^{**}} < \sqrt{e_1^{*}/e_2^{*}}$, *implying that quantity commitments reduce effort differentials.*

Proof. (24)-(25)are derived by substituting the optimal levels of output in the first order conditions for the effort levels. Solving the system one obtains the effort levels:

$$\sqrt{e_k^{**}} = \frac{(a-c)(12\theta_j-3)}{(12\theta_2-\nu-2)(12\theta_1+(1-\nu)-2)-\nu(1-\nu)} \text{ for } k,j=1,2$$

It is then possible to compute $\sqrt{e_1^{**}/e_2^{**}} = \frac{12\theta_2 - 3}{12\theta_1 - 3}$ that is unambiguously smaller than $\sqrt{e_1^{*}/e_2^{*}} = \frac{9\theta_2 - 4}{9\theta_1 - 4}$. ■

In other words, when contracts are more general and include quantity commitments the principal finds it optimal to induce lower effort for both efficient and inefficient agents. Of course, the lower effort levels tend to reduce production and increase profits. The intuition for these results is once again to be found in the fact that in our basic two stage setup firms tend to invest too much to commit to a higher production in the market. Since managers decide how much to produce without taking in consideration the impact on the rival, this leads to excessive investment *ex ante* and excessive production *ex post* from the point of view of the firms. The more general contract allows firms to limit this tendency and reduce final production.

In spite of these differences in our first best results, the introduction of asymmetric information determines the same qualitative results of our basic model. Once each agent has private information about his type when contracting with his own principal the problem of firm i can be stated as follows:

$$\max_{(e_k^i, x_k^i, w_k^i)} E(\pi_k^i) = \nu \left[\left(a - c + \sqrt{e_1^i} - x_1^i - \nu \left(x_1^j \right) - (1-\nu) \left(x_2^j \right) \right) x_1^i - w_1^i \right] + \tag{26}$$
$$+ (1-\nu) \left[\left(a - c + \sqrt{e_2^i} - x_2^i - \nu \left(x_1^j \right) - (1-\nu) \left(x_2^j \right) \right) x_2^i - w_2^i \right]$$

under the two standard binding (participation and incentive compatibility) constraints:

$$w_2^i = \theta_2 e_2^i, \;\; w_1^i = \theta_1 e_1^i + \Delta\theta e_2^i$$

The following Proposition summarizes our results for the case of quantity commitments in presence of asymmetric information.

Proposition 5 *When a contract includes output decisions and principals do not observe the type of their own manager the optimal level of efforts are given by the following equilibrium conditions:*

$$\sqrt{e_1^q} = \frac{2(a-c) - (1-\nu)\sqrt{e_2^q}}{12\theta_1 - 2 - (1-\nu)} \tag{27}$$

$$\sqrt{e_2^q} = \frac{2(a-c) - \nu\sqrt{e_1^q}}{12\left(\theta_2 + \frac{v}{1-v}\Delta\theta\right) - 2 - \nu} \tag{28}$$

Moreover $e_1^q > e_1^{**}$ *and* $e_2^q < e_2^{**}$*, meaning that both types exert inefficient levels of efforts.*

Proof. The first order conditions for the problem in question are:

$x_1^i = \frac{(a-c)+\sqrt{e_1^i}-\nu\left(x_1^j\right)-(1-\nu)\left(x_2^j\right)}{2}$

$x_2^i = \frac{(a-c)+\sqrt{e_2^i}-\nu\left(x_1^j\right)-(1-\nu)\left(x_2^j\right)}{2}$

$\sqrt{e_1^i} = \frac{x_1^i}{2\theta_1}$

$\sqrt{e_2^i} = \frac{x_2^i}{2\left(\theta_2+\frac{\nu}{1-\nu}\Delta\theta\right)}$

Once we impose the conditions for a symmetric equilibrium: $e_1^i = e_1^j = e_1$, $e_2^i = e_2^j = e_2$, $x_1^i = x_1^j = x_1$and $x_2^i = x_2^j = x_2$ we can find the optimal quantities that are the same as (22) and (23). If we substitute them in the FOC with respect to the effort levels we obtain the equilibrium conditions (27)-(28) above. Solving which we find:

$$\sqrt{e_k^q} = \frac{(a-c)\left(12\tilde{\theta}_z - 3\right)}{\left(12\tilde{\theta}_2 - \nu - 2\right)\left(12\tilde{\theta}_1 + (1-\nu) - 2\right) - \nu(1-\nu)} \text{ for } k, z = 1, 2 \text{ and } k \neq z$$

where $\tilde{\theta}_1 = \theta_1$ and $\tilde{\theta}_2 = \theta_2 + \frac{\nu}{1-\nu}\Delta\theta$. Since the virtual type of an inefficient manager is now higher than his true type it is immediate to verify that $e_1^q > e_1^{**}$ and $e_2^q < e_2^{**}$. ■

We have seen that the equilibrium screening contracts are characterized once more by two way distortions that imply a lower effort of the inefficient managers and a higher effort for efficient ones if compared to the first best case with symmetric information. As in the baseline model, the no-distortion at the top property disappears. We can also make clear statements on the efficiency of the firm once strategic contract offers happen in a one stage framework. Since the first best levels of effort were also those that minimized total costs for any level of output, our two way distortion necessarily brings down the efficiency of each firm, when a hierarchy employs an efficient manager he will be induced to exert too much effort while in the case of an inefficient manager effort will be too little.

8 Extensions

In this section we briefly examine other forms of competition to verify when this leads to a two-way distortion. As we will see, this depends on whether the effort levels are strategic complements or substitutes.

8.1 Hotelling competition

First, let us consider the case of price competition in a Hotelling model. We assume firms to be located at both ends of the unit segment and consumers, who are uniformly distributed along this segment, to have utility:

$$U = \max_i (1 - p_i - d_i) \quad i = 1, 2 \tag{29}$$

where d_i is the distance from producer i and p_i is the price charged by firm i for a unit of the good. Each firm has marginal cost $c - \sqrt{e^i}$ that depends on the effort of its manager.

At the second stage, once costs are realized and known to everybody, firm i has demand $D_i = (1 + p_j - p_i)/2$ and sets prices to maximize its profits:

$$\pi^i = \left(p_i - c + \sqrt{e^i}\right) \frac{(1 + p_j - p_i)}{2} - w^i \tag{30}$$

taking as given the price choice of the rival, and analogously for firm j. The equilibrium prices are:

$$p_i = 1 + c - \frac{2\sqrt{e^i} + \sqrt{e^j}}{3} \quad i, j = 1, 2 \tag{31}$$

This strategic behavior leads to the equilibrium profits $\pi^i = \Pi(e^i, e^j) - w^i$ with:

$$\Pi(e^i, e^j) = \frac{1}{2}\left(1 + \frac{\sqrt{e^i} - \sqrt{e^j}}{3}\right)^2 \tag{32}$$

where the effort levels are strategic substitutes:

$$\Pi_{12}(e^i, e^j) = \frac{-1}{12\sqrt{e^i e^j}} < 0 \tag{33}$$

as in our basic model with quantity competition and substitute goods. In the symmetric information setup, the expected profits of firm i with a manager of type $k = 1, 2$ can be expressed as the weighted average of the profits obtained when facing an efficient or an inefficient rival:

$$E(\pi^i_k) = \nu \frac{1}{2}\left(1 + \frac{\sqrt{e^i_k} - \sqrt{e^j_1}}{3}\right)^2 + (1 - \nu)\frac{1}{2}\left(1 + \frac{\sqrt{e^i_k} - \sqrt{e^j_2}}{3}\right)^2 - w^i_k \tag{34}$$

where (e^i_k, w^i_k) is the contract chosen by firm i when its manager is of type θ_k with $k =$

1, 2. The optimal contract maximizes the above expression subject to the participation constraint $w_k^i = \theta_k e_k^i$.The first-order conditions are:

$$\sqrt{e_k^i} = \frac{3 - \nu\sqrt{e_1^j} - (1-\nu)\sqrt{e_2^j}}{18\theta_k - 1} \text{ for } k = 1, 2$$

We can then impose the conditions for a symmetric equilibrium, $e_1^j = e_1^i$ and $e_2^j = e_2^i$, and derive the following equilibrium conditions:

$$\sqrt{e_1^{h*}} = \frac{1 - (1-\nu)\sqrt{e_2^{h*}}}{18\theta_1 - (1-\nu)} \tag{35}$$

$$\sqrt{e_2^{h*}} = \frac{1 - \nu\sqrt{e_1^{h*}}}{18\theta_2 - \nu} \tag{36}$$

These two expressions clearly show that the interdependence between the two levels of effort is the same as in our baseline model. It is then straightforward to notice that the impact of asymmetric information is also the same as before. The downward distortion on the inefficient type's effort leads to divergence of the effort levels, with extra effort for the most productive type and an additional downward distortion for the least productive types if compared to the first-best values determined above. The equilibrium efforts satisfy:

$$\sqrt{e_1^h} = \frac{1 - (1-\nu)\sqrt{e_2^h}}{18\theta_1 - (1-\nu)} \tag{37}$$

$$\sqrt{e_2^h} = \frac{1 - \nu\sqrt{e_1^h}}{2\left(\theta_2 + \frac{\nu}{1-\nu}\Delta\theta\right) - v} \tag{38}$$

Therefore, we can suggest that the type of competition in the market does not affect the general features of the equilibrium contracts and still present the two way distortion feature of our main example, what is relevant is how the effort of the other firm's manager affects the expected profit function.

8.2 Cournot competition with complement goods

We now consider the case of quantity competition with complement goods. Assume an inverse demand function for firm i given by:

$$p_i = a - x_i + bx_j$$

where b parametrizes complementarity. The Cournot equilibrium at the second stage when the two firms have managers who exert efforts e^i and e^j prescribes that each firm i produces the following output:

$$x_i = \frac{(2+b)(a-c) + 2\sqrt{e^i} + b\sqrt{e^j}}{(2-b)(2+b)} \quad \text{for } i,j = 1,2 \text{ and } i \neq j \tag{39}$$

that generates the second stage profit function below $\pi^i = \Pi(e^i, e^j) - w^i$ with:

$$\Pi(e^i, e^j) = \left[\frac{(2+b)(a-c) + 2\sqrt{e^i} + b\sqrt{e^j}}{(2-b)(2+b)}\right]^2 \quad \text{for } i,j = 1,2 \text{ and } i \neq j \tag{40}$$

The above equation shows that in this case we have strategic complementarity between efforts and each firm's profits depend positively from both efforts.

When each principal observes the type of his own manager, the expected profits of firm i with a manager of type $k = 1,2$ can be expressed as the weighted average of the profits obtained when facing an efficient or an inefficient rival:

$$E(\pi_k^i) = \nu \Pi(e_k^i, e_1^j) + (1-\nu)\,\Pi(e_k^i, e_2^j) - w_k^i \tag{41}$$

where (e_k^i, w_k^i) is the contract chosen by firm i when its manager is of type θ_k with $k = 1,2$. The optimal contract maximizes expected profits subject to the participation constraints $w_k^i = \theta_k e_k^i$.

The optimal effort of a manager of type θ_k in firm i satisfies the following first order condition:

$$\sqrt{e_k^i} = \frac{2(2+b)(a-c) + 2\nu b\sqrt{e_1^j} + 2(1-\nu)\, b\sqrt{e_2^j}}{\theta_k^i\left[(2-b)(2+b)\right]^2 - 4} \tag{42}$$

for $k = 1,2$.

In presence of asymmetric information regarding his own manager's type, each principal will instead maximize the following objective function:

$$\max_{\left(e_k^i, w_k^i\right)} \nu^2 \Pi(e_1^i, e_1^j) + \nu\,(1-\nu)\,\Pi(e_1^i, e_2^j) + (1-\nu)\nu\Pi(e_2^i, e_1^j) +$$

$$(1-\nu)^2\,\Pi(e_2^i, e_2^j) - \nu\left(\theta_1 e_1^i + \Delta e_2^i\right) - (1-\nu)\,\theta_2 e_2^i$$

$$\text{s.t. } w_2^i = \theta_2 e_2^i,\;\; w_1^i = \theta_1 e_1^i + \Delta\theta e_2^i$$

that is expected profit across four states of the world subject to the individual rationality and incentive compatibility constraints for the manager as seen previously.

In this case, when products traded at the second stage are complement goods, the optimal level of efforts are given by the following equilibrium conditions:

$$\sqrt{e_1^{c*}} = \frac{2\,(2+b)\,(a-c) + 2\,(1-\nu)\,b\sqrt{e_2^{c*}}}{\tilde{\theta}_1\left[(2-b)(2+b)\right]^2 - 4 - 2vb} \tag{43}$$

$$\sqrt{e_2^{c*}} = \frac{2\,(2+b)\,(a-c) + 2\nu b\sqrt{e_1^{c*}}}{\tilde{\theta}_2\left[(2-b)(2+b)\right]^2 - 4 - 2(1-v)b} \tag{44}$$

where $\tilde{\theta}_k$ is the virtual type of a manager of type k. When the type of each manager is not observed by his principal $\tilde{\theta}_2 = \theta_2 + \frac{v}{1-v}\Delta\theta$, and the level of effort is lower than in the symmetric information case: $\sqrt{e_2^c} < \sqrt{e_2^{*c}}$. The level of effort required from an efficient manager is also lower: $\sqrt{e_1^c} < \sqrt{e_1^{*c}}$.

To verify this, notice that taking the first order conditions (42) and imposing symmetry conditions on level of efforts allows us to derive the equilibrium conditions above. In the asymmetric information case an informational rent has to be paid to the efficient manager and it distorts downward the level of effort of the inefficient manager. The positive relation between the two level of efforts implies that also the effort for the efficient manager will be distorted downward.

The key aspect of this model is that, contrary to the previous models, the effort levels exerted by the managers in their own firm turn out to be strategic complements:

$$\Pi_{12}(e^i, e^j) = \frac{b}{(4-b^2)\sqrt{e^i e^j}} > 0 \tag{45}$$

This means that a higher effort of one manager increases the marginal profitability of the other firms. In such a case, both firms tend to implement more similar contracts (there is less to gain from effort from a good manager that is more likely to meet a bad one) with low effort, which softens competition increasing costs and profits. We do not observe two way distortions in this setup, nonetheless the no-distortion at the top property is absent also in this case.

The result that complement goods somehow reversed the direction of the strategic effect is present (although in different ways) also in most of the previous work that studied strategic contract offers as, for example, Martimort [1996], Brainard and Martimort [1996] and in a two stages setup Brander and Spencer [1983].

8.3 Contract competition with advertising effort

Consider the same market as in the baseline example where effort produces demand enhancing activities, as advertising, that increase total demand and in particular the demand of the firm investing in these activities. In particular, assume an inverse demand:

$$p_i = a + \sqrt{e_i} + b\sqrt{e_j} - X$$

where X is total quantity and $b < 1$. Given two contracts (e^i, w^i) and (e^j, w^j), firm i obtains the profits:

$$\pi^i = (a + \sqrt{e_i} + b\sqrt{e_j} - x_i - x_j)x_i - cx_i - w^i \tag{46}$$

where c is the now constant marginal cost, e_i and e_j represent the amount of advertising. The idea is that both types of advertising increase demand, but own advertising has a stronger effect than the rival's one.

At the second stage, once advertising investments have been made and are public, firms set quantities taking as given the quantity of the other firm. The optimal quantity choice is given by:

$$x_i = \frac{a - c + (2-b)\sqrt{e_i} + (2b-1)\sqrt{e_j}}{3} \text{ for } i, j = 1, 2 \text{ and } i \neq j$$

As a consequence the first stage profits, as a function of advertising efforts, become $\pi^i = \Pi(e^i, e^j) - w^i$ with::

$$\Pi(e^i, e^j) = \left[\frac{1}{3}\left(a - c + (2-b)\sqrt{e_i} + (2b-1)\sqrt{e_j}\right)\right]^2$$

this expression depends positively in both firm's advertising effort, meaning that we are facing another case of strategic complements as in our previous example of complement goods:

$$\Pi_{12}(e^i, e^j) = \frac{(2-b)(2b-1)}{6\sqrt{e_i e_j}} > 0 \tag{47}$$

In the symmetric information setup, the expected profits of firm i with a manager of type $k = 1, 2$ can be expressed as the weighted average of the profits obtained when facing an efficient or an inefficient rival:

$$\begin{aligned} E(\pi_k^i) &= \nu \left[\frac{1}{3} \left(a - c + (2-b)\sqrt{e_k^i} + (2b-1)\sqrt{e_1^j} \right) \right]^2 \\ &\quad + (1-\nu) \left[\frac{1}{3} \left(a - c + (2-b)\sqrt{e_k^i} + (2b-1)\sqrt{e_2^j} \right) \right]^2 - w_k^i \end{aligned} \tag{48}$$

where (e_k^i, w_k^i) is the contract chosen by firm i when its manager is of type θ_k with $k = 1, 2$. The optimal contract maximizes the above expression subject to the participation constraint $w_k^i = \theta_k e_k^i$.The first-order conditions are:

$$\sqrt{e_k^i} = \frac{(2-b)(a-c) + (2-b)(2b-1)\left[\nu\sqrt{e_1^j} + (1-\nu)\sqrt{e_2^j}\right]}{9\theta_k - (2-b)^2} \text{ for } k = 1, 2 \tag{49}$$

We are again looking for symmetric equilibrium so we can restrict attention to the situations where $e_1^j = e_1^i$ and $e_2^j = e_2^i$, and derive the following equilibrium conditions:

$$\sqrt{e_1^{a*}} = \frac{(2-b)(a-c) + (2-b)(2b-1)(1-\nu)\sqrt{e_2^{a*}}}{\left[9\theta_1 - (2-b)^2 - (2-b)(2b-1)\nu\right]} \tag{50}$$

$$\sqrt{e_2^{a*}} = \frac{(2-b)(a-c) + (2-b)(2b-1)\nu\sqrt{e_1^{a*}}}{\left[9\theta_2 - (2-b)^2 - (2-b)(2b-1)(1-\nu)\right]} \tag{51}$$

These conditions show that the first-best effort levels in our model with demand enhancing advertising are interdependent like in our example with complement goods, the amount of advertising required from an efficient manager depend positively on the effort required from an inefficient one.

Once we introduce asymmetric information inside the hierarchy, the downward distortion in the inefficient manager's effort due to the informational rent that has to be paid to the efficient type will cause a downward distortion in the more efficient manager's effort. The new equilibrium conditions are:

$$\sqrt{e_1^a} = \frac{(2-b)(a-c) + (2-b)(2b-1)(1-\nu)\sqrt{e_2^a}}{\left[9\theta_1 - (2-b)^2 - (2-b)(2b-1)\nu\right]} \tag{52}$$

$$\sqrt{e_2^{a*}} = \frac{(2-b)(a-c) + (2-b)(2b-1)\nu\sqrt{e_1^{a*}}}{\left[9\left(\theta_2 + \frac{v}{1-v}\Delta\theta\right) - (2-b)^2 - (2-b)(2b-1)(1-\nu)\right]} \tag{53}$$

and it is immediate to see that, although the no distortion at the top property is still not present, both efforts are downward distorted.

In conclusion, whenever efforts are strategic complements their equilibrium levels will be downward distorted with respect to the first best for all types. Therefore, the kind of strategic interactions between the efforts of the managers (and not the kind of competition) leads to different consequences on the equilibrium contracts.

9 Conclusions

In this work we have analyzed the choice of incentive contracts by firms that operated in an imperfectly competitive product market. The main result is that, due to a strategic effect in contract offers, the no distortion at the top, present in standard screening models, disappears. A two way distortion becomes optimal in our main model, when types are correlated, when the game is one stage and when there is spatial competition, all situations in which agent's efforts are strategic substitutes from the point of view of the principal. When efforts are strategic complements their optimal level is always downward distorted, even for the highest type.

We believe our work has offered two novel insights. First, we have contributed to the literature on competing hierarchies in identifying a new channel through which competition may affect incentive provision. Second, we have contributed to the more general contract theory literature in showing a new reason, beyond countervailing incentives and production externalities, for having a two way distortion.

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